Maximum Stable Droplets in Dispersoids

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Maximum stable droplet size in nonuniform dispersoids may be ignored or treated as a statistical-limiting parameter or as a physically significant quantity. In this paper the author shows that treatment as a physically significant quantity is always permissible and in most cases advantageous. It is found that maximum stable droplet size as well as mean droplet size can be calculated in some cases from theory and in other cases from correlated experimental data; this is illustrated with examples involving sprays and emulsions.

The importance of knowing something about particle sizes in dispersoids has often been noted (*4, 5, 15, 20*). But the importance of recognizing a maximum particle size is frequently overlooked; therefore, some evidence is presented that the acceptance of a maximum particle size is always permissible and in most cases advantageous. On the other hand the author indicates how the acceptance of a negligible fraction of infinitely large particles may be misleading, giving wrong answers in certain theoretical calculations and process evaluations. In this paper particles are considered which are liquid droplets. In this case the maximum stable size (*MSD*) is such that it may not be exceeded by any droplet which is in equilibrium with the environment. Methods are proposed for calculating the maximum stable diameter, in some cases from theory, and in other cases from correlated experimental data.

BASIC CONCEPTS: SIZE DISTRIBUTION, STABILITY, MAXIMUM STABLE DROPLET

The distribution of particle sizes in dispersoids has been treated in various ways by various authors. We are only indirectly concerned here with the relative merits of these treatments. It has been found that a Gaussian distribution of a certain parameter y adequately represents size distributions of many dispersoids (*17*). This distribution is assumed to prevail whenever no other is specified.

The parameter y nas the form $\ln a(x^s - x_o^s)/(x_m^s - x^s)$. Here a is a dimensionless skewness parameter ($a=1$ for symmetric distributions.) The exponent s is related to the manner of dispersion but is not especially critical in fitting the distribution to data. For the examples treated in this paper, $s=1$. The variable x is a linear measure of particle size (usually diameter, although any dimension proportional to diameter will do). The particular values x_o and x_m apply to the smallest and largest stable particle size that may exist in the dispersoid.

In some situations one is concerned with the minimum particle size, x_o. However these situations are rare, and for practical purposes x_o can usually be taken as zero (an approximation for a molecular diameter when compared with droplet sizes in ordinary aerosols, sprays, and emulsions).

However no such simple approximation is available for the maximum particle size. By analogy to the approximation $x_o = 0$, one might say $x_m = \infty$, but this can be quite misleading. More rationally one can think of a dispersoid in its proper environment and calculate (either from correlation or theory) a maximum stable size, using this as an approximation for x_m. As an example of such a calculation an idealized situation wherein a droplet is acted on only by the drag of a continuous medium is considered. When this drag overcomes the molecular force field at the interface, the droplet becomes unstable and bursts. Thus its maximum stable size is governed by the force balance

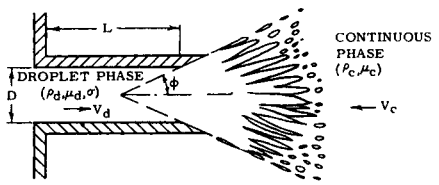

Fig. 1. Atomization variables (schematic).

$$C_D \cdot \frac{1}{2}\rho_c V^2 \cdot \frac{\pi}{4} x_m^2 = \pi x_m \cdot \sigma \quad (1)$$

Hence

$$x_m = 8\sigma/C_D\rho_c V^2 \quad (2)$$

This relation may also be expressed by the definition of a critical Weber number:

$$We_m = x_m\rho_c V^2/\sigma = 8/C_D \quad (3)$$

In the following sections the processes of mechanical atomization and mechanical emulsification are examined. Correlations for size of the mean droplet in a spray and also for the maximum stable droplet are developed. For completeness in representing the size distribution, values of the sharpness parameter are also deduced for some types of atomizers.

THE PROCESS OF ATOMIZATION

The term *atomization* is used in its usual sense of breakup of bulk liquid into droplets. Mechanical atomization is achieved in many different ways. Among the devices commonly employed are pressure nozzles, Venturi or gas-atomizing nozzles, rotating cups or disks, and bubble caps or grids or perforated plates.

Although details of the drop forming process vary widely among the different types of apparatus, certain events normally occur in all of these processes:

1. The applied force, whether due to pressure or rotation, separates protuberances, filaments, or sheets of liquid from the bulk liquid.
2. These protuberances, filaments, or sheets are broken up (owing to their inherent instability and to interaction with a mass of gas or liquid) into small particles.
3. These particles assume spherical shape if they are small enough. Or, if large enough to be unstable, they break

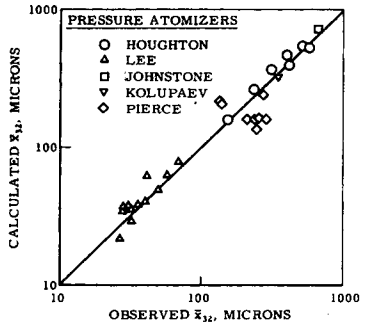

Fig. 2. Comparison of experimental and calculated values of $\bar{x}_{32}$ (pressure nozzles).

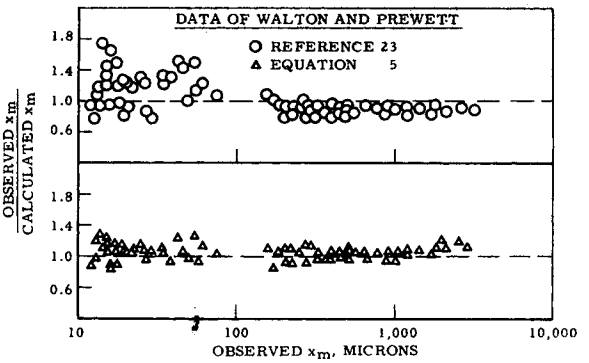

Fig. 4. Comparison of experimental and calculated values of x_m (spinning disks).

into smaller masses which then become spherical.

A detailed treatment of the motions of droplets is not part of the present subject but may be found in references *10* and *1*.

FUNDAMENTAL VARIABLES IN ATOMIZATION

The average size of droplets in a spray, as well as the distribution of droplet sizes, will depend on certain physical properties of the fluids involved, on design of the atomizer, and on operating conditions. The relevant physical properties include ρ_d *and* ρ_c, μ_d and μ_c, and σ.

The relevant design variables are not easily stated in general terms because of the wide variety of spray generators to be considered. For the majority of atomizers D, L, and ϕ will suffice.

The relevant operating conditions again will depend on the particular type of atomizer used. For example the important variables may appear as pressures (for a pressure nozzle), volume flow rates (for a Venturi nozzle), or angular velocity (for a rotating cup). But generally by the use of mechanical and hydrodynamic relations these may be reduced to V_d and V_c, or V, and β. Fundamental variables are illustrated schematically in Figure 1.

GENERAL CORRELATING EQUATION FOR ATOMIZATION

Having considered the fundamental variables which determine the droplet-size distribution in atomization one proceeds to investigate their relation to the representative droplet size in the resulting spray.

Dimensional analysis indicates that the following dimensionless groups are sufficient to describe this relation: the size ratios $\bar{x}/D$ and $\bar{x}/L$ and the design angle ϕ, the Froude number $V^2/D\beta$, the Reynolds numbers $D\rho_c V_c/\mu_c$ and $D\rho_d V_c/\mu_d$, the interfacial-tension groups $\mu_c V_c/\sigma$ *and* $\mu_d V_d/\sigma$, and the density ratio ρ_c/ρ_d.

A simple form in which the most general relation between these groups may be written is

$$f\left(\frac{\bar{x}}{D}, \frac{V_d^2}{D\beta}, \frac{L}{D}, \phi, \frac{D\rho_d V_d}{\mu_d}, \frac{\mu_d V_d}{\sigma}, \frac{\rho_c}{\rho_d}, \frac{\mu_c}{\mu_d}, \frac{V_c}{V_d}\right) = 0 \qquad (4)$$

In this equation the representative size may be one of the mean diameters (*17*), the median diameter, or the maximum diameter. The form of the function will vary according to which of these diameters one chooses. In general at least three independent diameters are needed to define the distribution completely (for example $\bar{x}_{32}$, x_m, and x_{50}). But if one has an independent method for determining one of the distribution parameters, two of the diameters may suffice. In this paper $\bar{x}_{32}$ and x_m are individually correlated, and the distribution parameter δ [Equation (8) and reference *17*] is tabulated for certain types of atomizers. Concerning the distribution parameters, one notes that even more than three might sometimes be needed, because of the complex nature of the dispersion mechanism, not merely because of the form of the distribution equation.

CORRELATION OF MAXIMUM AND MEAN DROPLET SIZES FOR ATOMIZATION IN PRESENCE OF GASES

Equation (*4*) shows the dimensionless groups found to affect x_m or $\bar{x}_{32}$. Most atomization data have been obtained by dispersing liquid into gases not greatly different from atmospheric air. Thus the effect of continuous-phase properties (that is, the groups ρ_c/ρ_d and μ_c/μ_d) on drop size is less clearly defined than is the effect of other dimensionless groups involving only dispersed-phase properties. For this reason the correlations of x_m and $\bar{x}_{32}$ discussed

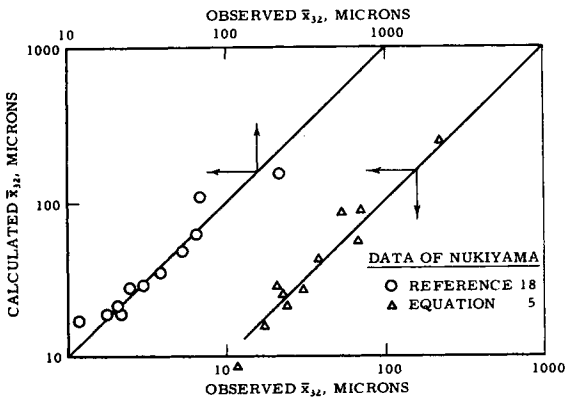

Fig. 3. Comparison of experimental and calculated values of $\bar{x}_{32}$ (Venturi atomizers).

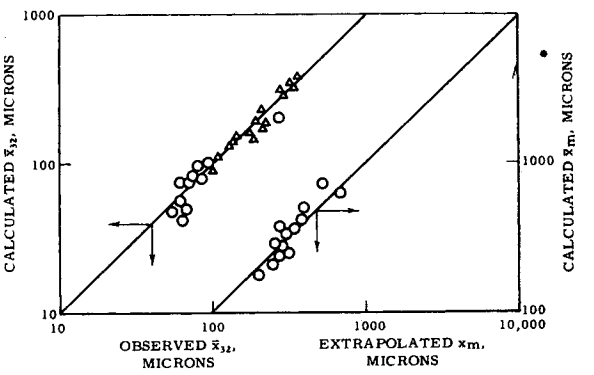

Fig. 5. Comparison of experimental and calculated values of $\bar{x}_{32}$ and x_m (effective diameter method).

TABLE 1. SUMMARY OF CONSTANTS TO BE USED IN ESTIMATING DROP SIZE DISTRIBUTION

Type of atomizer	Calculation of $\bar{x}_{32}$* A	B	C	Standard deviation, %	Calculation of x_m* A_m	B_m	C_m	Standard deviation, %
Straight pressure or** tangential nozzle	5.0	−0.35	−0.20	22	57	−0.48	−0.18	34
Venturi atomizer††	1140	−0.82	−0.45	30				
Modified pressure nozzles** (pintle, lip, impinging jet, etc.)†	5.0	−0.35	−0.20	30	57	−0.48	−0.18	22
Spinning disks or cups	—	—	—	—	1.73	−0.50	−0.45	15
Whirl chambers	5.0	−0.35	−0.20	26	57	−0.48	−0.18	42

* Constants A, B, and C for use in the equation $\frac{\bar{x}}{D} = A\left[\frac{D\rho_d V}{\mu_d}\right]^B\left[\frac{\mu_d V}{\sigma}\right]^C$.

** V_d calculated from Equation (6); D calculated from Equation (7).

† For pictures of these and other atomizers see references 5 and 9.

†† Values apply for Venturi atomizers only when volumetric ratio of liquid to vapor rate $Q_d/Q_c \leq 0.0001$.

in this paper are recommended only for atomization into gases having about the same viscosity and density as atmospheric air.

The few experimental data available indicate that the groups L/D and ϕ have little effect on drop sizes. These groups have been neglected in the present correlations. In addition, no detectable effect of Froude number was found in correlation of the data. After elimination of the above groups the following equation has been devised for the calculation of $\bar{x}_{32}$ or x_m for atomization into air:

$$\frac{\bar{x}}{D} = A\left[\frac{D\rho_d V}{\mu_d}\right]^B\left[\frac{\mu_c V}{\sigma}\right]^C \qquad (5)$$

It is interesting to note that the correlations or theories of Hinze (*6*), Milborn (*8*), and Walton and Prewett (*23*) also lead to these same dimensionless groups when continuous phase properties are neglected.

Constants for Equation (5) have been evaluated from experimental data for a number of different atomizer types and are shown in Table 1. Values of x_m used in obtaining the above correlation were calculated from distribution data by the use of the upper-limit method of Mugele and Evans (*17*). Fewer data are available for x_m than for $\bar{x}_{32}$, since some authors reported only $\bar{x}_{32}$ without giving the distribution data necessary to calculate x_m. For this reason correlation of x_m is considered less reliable than correlation of $\bar{x}_{32}$.

The relative velocity between phases, V rather than V_d, has been required for use in Equation (5) to correlate the experimental data. Relative velocity has also been used for correlation purposes by Merrington and Richardson (*16*) and Nukiyama and Tanasawa (*18*). For spinning atomizers the peripheral velocity $D\omega/2$ should be used for dispersed-phase velocity. For atomizer types other than those shown in Table 1, V_d can be calculated from

$$V_d = C_o\sqrt{2\Delta p/\rho_d} \qquad (6)$$

In this equation Δp is meant to be the pressure drop corresponding to energy which is directed toward breakup of the bulk liquid. If certain turns, constrictions, baffles, etc., produce friction before the liquid reaches the atomizing zone, the pressure losses so produced should be estimated and subtracted from the over-all pressure drop to determine Δp. On the other hand, if a whirl chamber or rotor contributes to turbulence in the bulk liquid and hence to instability which persists until breakup, then a corresponding pressure loss is properly part of Δp. Pressure losses across nozzles, as reported in the literature, are ordinarily close enough to Δp to be used without modification. There is some question about the use of the orifice velocity as in Equation (6), when actually the vena-contracta velocity may be more appropriate to conditions in jet breakup. Since this question has not been resolved with the available data, the best suggestion for the present is that V_d of Equation (6) be regarded as an effective velocity which merely interprets the effect of Δp.

If the effective discharge coefficient is unknown, a value of 0.7 is recommended. In cases where continuous-phase velocity is important, it must be estimated from flow or pressure-drop considerations. It should be noted that no continuous-phase velocity should be used when this velocity is created by flow of the dispersed phase, that is, by surface drag.

For spinning atomizers the representative diameter D to be used in Equation (5) is the outer diameter of the spinning part. For pressure or Venturi atomizers it is the orifice diameter (or hydraulic diameter) of the liquid disperser. If the geometry of the spray generator is complicated by interior grooves, vanes, multiple inlets, or pintles (or for the swirl type of atomizers which have an air cone), D cannot be calculated as readily. However it has been found that the effective diameters for use in Equation (5) can be calculated from Equation (7) if the volumetric flow and pressure drop are known:

$$D = \sqrt{4Q_d/\pi V_d} \qquad (7)$$

where V_d is calculated from Equation (6).

THE UNIFORMITY PARAMETER

The parameter δ is a measure of uniformity of the spray, a larger value of δ indicating a more uniform spray. Various attempts to correlate δ have failed to produce a general correlation with atomizer dimensions. Table 2 shows the range of values of δ found in various atomizer types together with recommended values. For further discussion of the parameter the reader should consult reference *17*. The defining equation for δ is

$$\frac{dv}{dy} = \frac{\delta}{\sqrt{\pi}}e^{-\delta^2 y^2} \qquad (8)$$

For atomizers not classified in Table 2 and for which specific data are not

TABLE 2

Atomizer type	Source of data	Range of δ	Recommended δ
Straight pressure nozzle	(*11*) (*9*)	0.618-0.670	0.64
Modified pressure nozzle (pintle, lip, impinging jet, etc.)	(*21*) (*13*)	0.861-1.108	0.98
Tangential nozzle (Spraco-T)	(*9*)	0.600-0.750	0.68
Rotating-cup atomizer	(*8*)	0.71 -1.11	1.00
Whirl-chamber atomizer	(*12*) (*19*)	1.134-1.145	1.14
Venturi atomizer (gas injection)	(*14*) (*3*)	1.03 -1.52	1.28

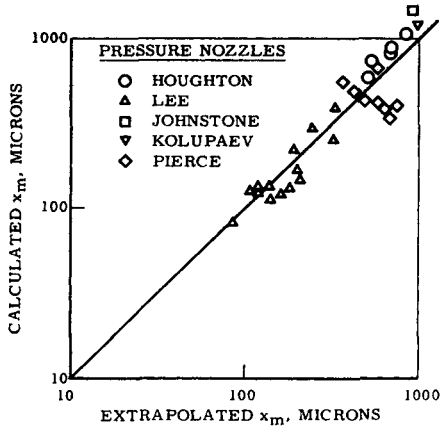

Fig. 6. Comparison of extrapolated and calculated values of x_m (pressure nozzles).

available the value $\delta = 1.00$ is suggested.

COMPARISON WITH EXPERIMENTAL DATA

Values of x_m and $\overline{x_{32}}$ calculated from Equation (5) by the use of values of constants shown in Table 1 are compared with experimental data in Figures 2 to 6. Some of these figures also show comparisons with other correlations. Standard deviations of data from correlation are included in Table 1. No general advantage is claimed for this correlation method over others. The author is aware of many recent and excellent presentations of spray data and correlations of various kinds, including those of references *15* and *20*. The calculations that have been presented here were made before the spray data became plentiful and will be revised to do justice to the recent data. However the foregoing calculations have been presented to illustrate the efficacy of separately correlating a mean droplet size and a maximum stable droplet size.

APPLICATION OF THE CORRELATIONS

Values of x_m, $\overline{x_{32}}$, and δ estimated from the correlations in this paper may be used to calculate the complete drop-size distribution resulting from various atomization conditions when continuous-phase viscosity and density do not differ appreciably from the values for atmospheric air. Sample calculations showing the application of the upper-limit method for calculating the complete distribution are shown later.

In many instances the complete drop-size distribution is not required, and the correlations may merely be used to estimate the trend of $\overline{x_{32}}$ or x_m with physical properties, apparatus design, or flow conditions. Thus even though absolute magnitude of predicted drop size may be in error, it is still possible to evaluate the effect of condition changes on drop size. This information is frequently useful in relating pilot-plant studies to large-scale performance.

Although the correlations have been derived for relatively simple atomizer types, it is believed that they will prove useful in estimating magnitude or trends of drop size in more complex apparatus. As was pointed out in a previous paper on this subject, such estimates are necessary for fundamental analysis of heat or mass transfer or of the separation of phases in a dispersed system. As an example of such an application the author has analyzed a photograph of the spray over a 3- by 3-ft. experimental grid tray having ¾-in. full round bars with ¼-in. spacing. The water rate was 5 gal./min. and the air rate 2,500 cu. ft./min. This photograph was taken at a 2-mm. depth of focus about 1in. inside the glass wall of the column. Drop sizes for this spray were counted by projecting the negative by a photographic enlarger onto a white blotter at magnification of 10. The only images suitable for measurement were those in sharp focus, that is, spherical images showing high lights. The following table shows a comparison of experimentally determined x_m and $\overline{x_{32}}$ with those calculated from Equation (5), the hydraulic diameter ($2w_s$) being substituted for D and constants shown in Table 1 for straight pressure nozzles used.

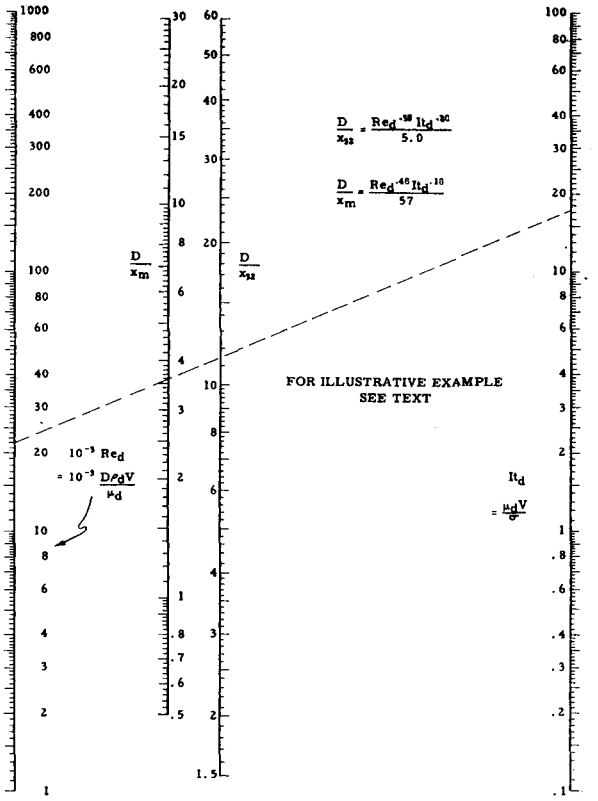

Fig. 7. Nomograph for droplet sizes, atomization via pressure nozzles into gases similar to atmospheric air.

	Calculated	Experimental
x_m, cm.	0.27	0.22 (largest drop measured)
$\overline{x_{32}}$, cm.	0.12	0.12

The agreement between experimental and calculated values is considerably better than could be expected in view of the uncertainties in interpreting the photograph. However it indicates that the correlation methods yield the correct order of magnitude.

The reader should be cautioned not to regard the conditions above as typical of grid-tray operations. The liquid rate was intentionally kept low to reduce the spray density.

ESTIMATION OF DROPLET-SIZE DISTRIBUTIONS WITH THE MAXIMUM STABLE DROPLET SIZE

Figure 7 shows a nomograph which can be used for rapid estimates of x_m or $\overline{x_{32}}$ as functions of the interfacial tension and Reynolds number groups for straight or modified pressure nozzles and whirl chambers. This nomograph applies only to atomization into gases similar to atmospheric air. Similar nomographs could be constructed for other atomizers by the use of the constants given in Table 1. One should note that $\overline{x_m}$ as well as $\overline{x_{32}}$ can be readily evaluated by using this nomograph. This application is illustrated in the following example.

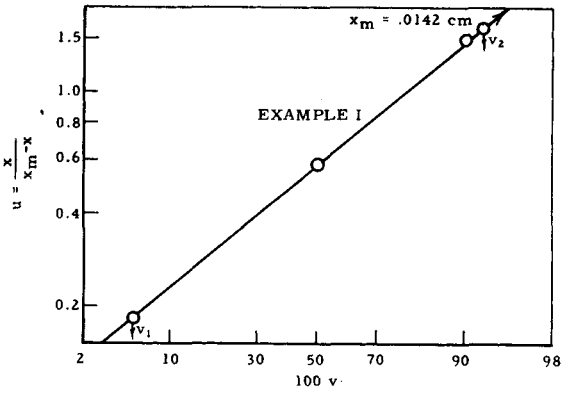

Fig. 8. Example of estimated droplet-size distribution.

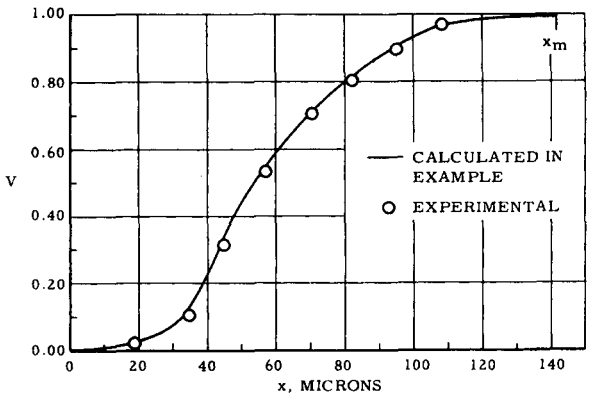

Fig. 9. Comparison of estimated distribution with data (example).

Example

Consider an oil of density $\rho_d = 0.86$ g./cc., viscosity $\mu_d = 0.0311$ poise, surface tension $\sigma = 28$ dynes/cm. Let it be injected into slightly compressed air (density 0.0015 g./cc.) via a modified pressure nozzle with diameter $D = 0.0508$ cm., with a pressure drop $\Delta p = 286 \times 10^6$ dynes/sq. cm. across the nozzle and a discharge coefficient of 0.6. One wishes to estimate the mean droplet size $\bar{x}_{32}$ in the resulting spray, also the volume percentage of spray which appears in droplets of diameter less than $\bar{x}_{32}/2$ and in droplets greater than $2\,\bar{x}_{32}$.

First one evaluates V.

$$V \cong 0.60\sqrt{2\Delta p/\rho_d}$$

$$= 0.60\sqrt{2 \times 286 \times 10^6/0.86}$$

$$= 15{,}500 \text{ cm./sec.} \quad (9)$$

Hence

$$\frac{D\rho_d V}{\mu_d} = 21{,}800; \quad \frac{\mu_d V}{\sigma} = 17.2 \quad (10)$$

From the nomograph (Figure 7)

$$\frac{D}{\bar{x}_{32}} = 11.5 \quad (11)$$

Therefore

$$\bar{x}_{32} = 0.00442 \text{ cm.}$$

Also

$$\frac{D}{x_m} = 3.58 \quad (12)$$

Therefore

$$x_m = 0.0142 \text{ cm.} \quad (13)$$

To complete the distribution picture, one assumes $\delta = 0.98$ (Table 2). From Equation (10.1) of reference *17*

$$r_{32} = \frac{x_m}{\bar{x}_{32}} = 1 + a\,e^{1/4\delta^2} \quad (14)$$

Therefore

$$a = \left(\frac{0.0142}{0.00442} - 1\right) \Big/ e^{1/4(0.98)^2}$$

$$= \frac{2.21}{1.297} = 1.704 \quad (15)$$

$$\frac{x_{50}}{x_m - x_{50}} = u_{50} = \frac{1}{a} = 0.587 \quad (16)$$

From Equation (9.6) of reference *17*

$$\log \frac{u_{90}}{u_{50}} = \frac{0.394}{\delta} \quad (17)$$

Therefore

$$\frac{u_{90}}{u_{50}} = 2.53; \; u_{90} = 1.49 \quad (18)$$

From the values of u_{90} and u_{50} one can plot the distribution line (Figure 8).

For the last part of the problem, the droplet sizes in question are

$$\left.\begin{aligned} x_1 &= \bar{x}_{32}/2 = 0.00221 \text{ cm.} \\ \text{and} \\ x_2 &= 2\,\bar{x}_{32} = 0.00884 \text{ cm.} \end{aligned}\right\} \quad (19)$$

The corresponding u values are

$$u_1 = 0.184 \text{ and } u_2 = 1.65 \quad (20)$$

Reference to Figure 8 gives the corresponding values of v:

$$v_1 = 0.052 \text{ and } v_2 = 0.923 \quad (21)$$

Thus this estimate gives 5.2% of the spray volume in droplets of diameter less than $\bar{x}_{32}/2$ and 7.7% in droplets of diameter greater than $2\,\bar{x}_{32}$.

Since the data used in this example correspond to those of a particular experiment [Lee (*13*)], it is possible to compare estimated results with experimental ones (Figure 9). The value of $\bar{x}_{32}$ given by these data is 0.00452 cm. and the value of x_m 0.0140 cm.

ERRORS ARISING FROM EXTRAPOLATION BEYOND THE MAXIMUM STABLE DROPLET SIZE

Some attempts to extrapolate distribution data to infinite particle size lead to untenable results. For example, extrapolation based on the Nukiyama-Tanasawa equation sometimes leads to a calculated value of $\bar{x}_{32}$ larger than any experimentally observed droplet diameter. In this connection one observes a weakness of distribution equations of form

$$\frac{dn}{dx} = A_1 x^{A_2} \exp\,(-A_3 x^{A_4}) \quad (22)$$

The mean diameter is found to be

$$\bar{x}_{pq} = A_3^{-1/A_4}\left[\Gamma\left(\frac{A_2 + 1 + p}{A_4}\right)/\Gamma\left(\frac{A_2 + 1 + q}{A_4}\right)\right]^{1/(p-q)} \quad (23)$$

Thus, considering the properties of the gamma function, one finds that p and q can be chosen to produce arbitrarily large values of $\bar{x}_{pq}$, even with p and q in a reasonable range. For example, with $A_2 = -5$, $A_3 = \bar{x}_{32}$, and $A_4 = -1$ one finds that x_{p3} increases without bound as p approaches 4.

When they are anticipated, errors of this kind can be avoided by various devices. But a more subtle kind of error can arise in process calculations when the effect of extrapolation is less spectacular. For example if one is concerned with Langmuir evaporation of spray droplets, then (as shown in reference *17*) the evaporation rate is proportional to the mean diameter $\bar{x}_{10}$. If one assumes an unlimited log-normal distribution with parameter $\delta = 1$, the calculated mean is

$$\bar{x}_{10} = x_{50} \exp\,(-5/4\,\delta^2)$$

$$= x_{50}/3.48 \quad (24)$$

But if one uses an upper-limit distribution with $\delta = 1$ and $a = 1$ (symmetrical), the result is

$$\bar{x}_{10} = 2x_{50}\,\frac{1 + 2a\exp\,(1/4\,\delta^2) + a^2\exp\,(1/\delta^2)}{1 + 3a\exp\,(1/4\,\delta^2) + 3a^2\exp\,(1/\delta^2) + a^3\exp\,(9/4\,\delta^2)} \quad (25)$$

$$= x_{50}/1.79$$

Thus the assumption of unlimited droplet size may cause a significant underestimation of the evaporation rate.

Similar calculations can be made to illustrate the effects on other processes involving dispersoids. In each case one should use the mean diameter appropriate to the process. The following table illustrates some further comparisons of the kind calculated above:

Ratio	Calculated by Log normal	Upper limit
$\bar{x}_{32}/x_{50}$	0.780	0.875
$\bar{x}_{31}/x_{50}$	0.606	0.798
$\bar{x}_{30}/x_{50}$	0.472	0.707
$\bar{x}_{10}/x_{50}$	0.287	0.559
$\bar{x}_{32}/x_{10}$	2.72	1.57

As previously indicated in reference *17*, the maximum stable droplet size is of some importance in understanding and correlating the generation of liquid-liquid dispersions, that is unstable emulsions. This fact has been noted (*2, 7, 22*), and some theories have been advanced as to the maximum stable droplet size in such systems (*7, 22*).

A theoretical treatment which proves useful in correlating many of the liquid-liquid dispersion data is that of Hinze (*7*). According to this the maximum stable droplet size in an unstable emulsion may be calculated from

$$k_m = x_m \, (\rho_c/\sigma)^{3/5} \, \epsilon^{2/5} \qquad (26)$$

Examples similar to those exhibited for sprays can be constructed to show the essential role played by the maximum stable droplet sizes in calculating the behavior of dispersoids of this kind (continuous-phase liquid).

SUMMARY

As proposed at the start, reasons have been stated for giving careful consideration to the maximum stable droplet size in describing dispersoids. Evidence has been given to show that the maximum stable droplet diameter can be separately correlated for many dispersoids and then used in process calculations and that the omission of this parameter may be misleading.

ACKNOWLEDGMENT

The author wishes to thank the Shell Development Company for permission to publish this material, and also his numerous colleagues whose contributions to the field are inextricably woven into the material of this paper. Special thanks are due to Mott Souders and Harry Evans, who supervised the research summarized here.

NOTATION

a = parameter in distribution equation
A, B, C = correlation constants, Table 1
C_D = drag coefficient
C_o = effective discharge coefficient
D = representative dimension perpendicular to flow
e = natural constant (2.718. . .)
f = function
It = interfacial tension group, $\mu V/\sigma$
k = size parameters, miscellaneous
k_m = constant, Equation (26)
L = representative dimension parallel to flow
n = numerical fraction of spray in droplets of diameter less than x
p = liquid pressure
Δp = pressure drop effective in producing atomization
Q_c = volume flow rate of continuous phase
Q_d = volume flow rate of droplet phase
r_{32} = ratio of x_m to $\bar{x}_{32}$
Re = Reynolds number, $\rho VD/\mu$
s = dispersion parameter (See definition for y)
u = function of droplet size, $= x/(x_m - x)$
u_{50} = value of u at $v = 0.50$
u_{90} = value of u at $v = 0.90$
v = volume fraction of spray in droplets of diameter less than x
V = velocity of droplet phase relative to continuous phase
V_c = velocity of continuous phase relative to atomizer
V_d = velocity of droplet phase relative to atomizer
w_s = slot width in grid tray
We_m = critical Weber number, $x_m \rho_c V^2/\sigma$
x = droplet diameter
x_m = maximum stable droplet diameter
$\bar{x}$ = representative or average droplet size
$\bar{x}_{pq}$ = generalized mean diameter
$\bar{x}_{32}$ = surface-volume (Sauter) mean diameter
x_{50} = volume median diameter
y = a function of droplet diameter; $y = \ln a(x' - x_o')/(x_m' - x')$; y has the Gaussian distribution indicated in Equation (8)

Greek Letters

β = acceleration imparted to liquid by body forces
Γ = gamma function (generalized factorial)
δ = uniformity parameter in distribution equation
ϵ = rate of energy dissipation in turbulent flow (per unit mass)
μ_c = viscosity of continuous phase
μ_d = viscosity of droplet phase
π = natural constant (3.14. . .)
ρ_c = density of continuous phase
ρ_d = density of droplet phase
σ = interfacial tension
ϕ = representative angle for diverging outlet
ω = angular velocity (rotating cup or disk), radians/sec.

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